Eigenvalues of Tensors and Their Applications

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Abstract

A tensor is represented by a supermatrix under a co-ordininate system.

Motivated by the positive definiteness issue in automatic control, I defined eigenvalues and eigenvectors of a real completely symmetric supermatrix, and explored their practical applications in judging positive definiteness of an even degree multivariate form.

However, the tensor studied in nonlinear continuum mechanics and physics are physical quantities which are invariant under co-ordinate system changes. In particular, the coefficients of the characteristic polynomial of a second order tensor are principal invariants of that tensor. Motivated by this, Qi, Rogers and Schief defined E-eigenvalues and E-eigenvectors for tensors. The E-eigenvalues of a tensor are the same as the E-eigenvalues of its representation supermatrix in an orthonormal co-ordinate system. Based upon the resultant theory, we define the E-characteristic polynomial of an mth order n-dimensional tensor. A complex number is an E-eigenvalue of the tensor if and only if it is a root of the E-characteristic polynomial. We show that the coefficients of the E-characteristic polynomial are invariants of the tensor. Let $d = d(m, n)$ be the number of such invariants. We show that $d(1, n) = 1, d(2, n) = n, d(m, 2) = m$ and $d(m, n) \leq m^{n-1} + \cdots + m$ for $m, n \geq 3$.